coordinate produced by the test to that which would have been produced by the specification impulses is

$$\bar{q}_i^t/\bar{q}_i^s = Q_{i0}^t[\text{sum}]^t/Q_{i0}^s[\text{sum}]^s \quad (7)$$

Although the form of the generalized forces is unknown, since it is supposed that there are no spatial differences between the specified and test forcing functions, the ratio Q_{i0}^t/Q_{i0}^s equals the ratio of the maximum force or pressure of the test to that specified. These latter quantities are known. Denoting them by F^t and F^s, we have

$$\bar{q}_i^t/q_i^s = F^t[\text{sum}]^t/F^s[\text{sum}]^s \quad (8)$$

If Eq. (8) ≥ 1 for the ith mode, the test impulses generated more excursion in that mode than the specified impulses would have. If Eq. (8) ≥ 1 for all i, the test is clearly as severe as that specified. If Eq. (8) < 1 for all i, the test is clearly not as severe as that specified.

Discussion

For those cases when Eq. (8) ≥ 1 for some modes and <1 for others, judgement must still be applied. However, it is felt that knowing these ratios for the first few modes is a great advantage over guesswork. Perhaps a reasonable criterion would be that the average of Eq. (8) over the known modes be at least one.

Reference

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Sonic Boom Minimization Schemes

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RECENTLY, it has been suggested that desirable modifications of objectionable sonic boom pressure signatures may be accomplished by the addition of mass or energy or by electro-aerodynamic means. The possibility of successfully utilizing any of these schemes is a matter of much current controversy.[1-3] It is the purpose of this Note to present simple analytical techniques with which proposals involving mass or energy addition may be evaluated and to infer some preliminary results concerning their feasibility.

The objective of the proposed mass or energy addition schemes is to create a "phantom" boundary which will favorably alter the effective area distribution of a given airplane. Such favorable alterations may be designed to produce either plateau ($A_e\alpha X^{3/2}$) or finite rise time ($A_e\alpha X^{5/2}$) pressure signatures.[4] The effective area variation required of the addition scheme is, therefore, the difference in effective areas between the phantom and actual bodies

$$\Delta A(X) = A_{\text{PHANTOM}}(X) - A_{\text{ACTUAL}}(X) \quad (1)$$

Assuming for the time being that this distribution has been selected, the problem becomes one of relating the required area growth to a causal mass or energy distribution. Identifying the phantom boundary as the "dividing streamline" for cases involving mass injection only, the mass distribution required to produce a given variation in area under the flight conditions of interest here can be determined by application

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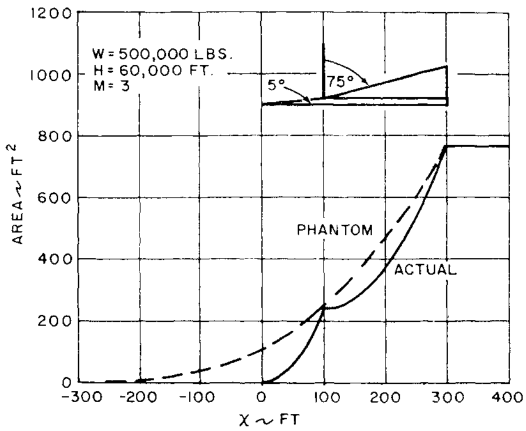

Fig. 1 Effective area distributions of actual configuration and phantom boundary.

of the results of slender body theory[5]

$$\dot{m}(x) = \rho_\infty U_\infty \int_0^x (1 + \tfrac{1}{2}M_\infty{}^2 Cp\delta t)\Delta A'(t)dt \quad (2)$$

where the pressure coefficient is explicitly related to the area distribution by Eq. (9.34C) of Ref. 5.

The analysis of energy addition schemes is more complex and the model adopted should, strictly speaking, be dependent upon the proposed manner in which the energy is to be added (conduction, convection, radiation). The problem may be viewed as essentially an inviscid interaction problem in which the flow within a reference streamtube tries to expand in area (due to heat addition) against a self-induced, retarding pressure gradient. (It is assumed that thermal layer growth ($\alpha X^{1/2}$ or $X^{0.8}$) will not be sufficiently rapid to be important in this application).

The inner (duct-like) flow can be described by the one-dimensional flow equations

$$(d/dx)(\rho u A) = 0 \quad (3a)$$

$$\rho u(du/dx) + (dP/dx) = 0 \quad (3b)$$

$$\frac{\gamma}{\gamma - 1}\rho u_A\left(\frac{1}{\rho}\frac{dP}{dx} - \frac{P}{\rho^2}\frac{d\rho}{dx} + \frac{\gamma - 1}{\gamma}u\frac{du}{dx}\right) = \dot{q}(x) \quad (3c)$$

Equations (3) are 3 equations involving 5 quantities. The additional relations required for closure of the system may be obtained from 1) the outer flow, where it is required that the axial pressure distribution and area variation of the reference streamtube be related[5] and 2) the requirement that the area variation be that given by Eq. (1). Cases involving both heat and mass addition would require the addition of a source term in the continuity equation plus one more relation (such as specifying either $\dot{m}$ or $\dot{q}$ and solving for the other or giving a functional relation between $\dot{m}$ and $\dot{q}$).

With these analytical models available, the mass or energy requirements to suitably modify the effective area distribution of a simple cone-cylinder-subsonic leading edge delta wing configuration can be estimated. The effective area of this configuration can be obtained analytically[6] and is shown in Fig. 1. (Wing-body interference effects were neglected and only normal "cutting planes" were considered.) A finite-rise-time bow shock modification requirement was postulated and the resulting minimum length phantom body effective area curve, also shown in Fig. 1, was found. This minimum length phantom was about 255 ft longer than the reference aircraft; shorter phantoms being prohibited since they resulted in negative areas for the axisymmetric "blown" body. The blown body area distribution, from Eq. (1), is given in Fig. 2. The mass addition analysis can be applied in an approximate manner (by neglecting the variation of density) and results in an approximate mass flow distribution given by

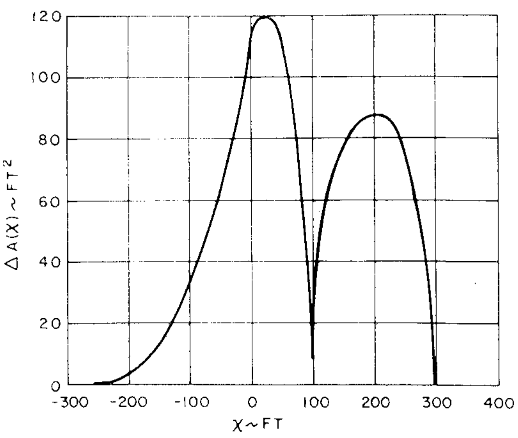

Fig. 2 Area distribution required of addition scheme.

$$\tilde{m}(x) = \rho_\infty U_\infty \Delta A(x) \quad (4)$$

Since the maximum area change required is about 120 ft², it is found that a maximum mass flow of approximately 2500 lb/sec is required. This estimate is conservative since the actual density contributions up to the body station where the maximum area change occurs will increase the result. This mass flow rate is about the total capability of the SST engines.[7] If the additional weight necessary to cycle this mass flow is estimated as that of the engines plus the additional fuel to run a second set of engines, the total weight penalty without considering the additional lift required is prohibitive even for a perfect system. In addition, if only 10 lb/sec were not reclaimed, it would result in a loss of over 100,000lb of material during the course of a three hour flight. The question of whether heat addition is possibly more effective can be answered by applying the analysis embodied in Eqs. (3) in an approximate manner. A "most favorable" estimate will be obtained by neglecting the pressure interaction since there will then be no induced counter force to oppose the area growth. The result of this procedure is

$$\int_{x_0}^{x} \dot{q}(t)dt = \frac{\gamma}{\gamma - 1}[P_\infty U_\infty(\Delta A(x) - A_0)] \quad (5)$$

where X_0 is the station at which $\Delta A(X_0) = A_0$.

Since the continuity and momentum equations result in the fact that ρA = const, reasonable upper limits of the area ratio can be deduced by imposing limits on the temperatures to be reached.† At an area ratio of $\Delta A_{max}/A_0 = 9$, one obtains a power level of $\frac{1}{3}$ million horsepower which is approximately 70% of the engine capability. The weight penalties are, therefore, essentially the same as in the mass addition case. If one attempts to produce "lower bound" shapes using these schemes the weight penalties increase. The energy required to obtain the maximum area change, ΔA_{max}, falls to zero as the reference streamtube's initial area approaches ΔA_{max}. In this case, there will be no area variation of the reference streamtube and consequently the required phantom boundary shape ($\alpha X^{5/2}$) will not be obtained. If large initial streamtube areas are to be considered, the final area change must be allowed to increase such that the proper distribution is attained. This will involve larger volumes of air and, hence, even more power than estimated previously.

In summary, simple techniques by which the effects of mass and energy addition upon the area distribution may be assessed have been presented. Also presented were the results of some sample calculations which indicate that the mass and energy addition schemes are probably not competitive with configurational changes as sonic boom minimization techniques.‡

† It is postulated that the transition from supersonic to subsonic flow is indicative of strong shock generation and should be avoided.

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‡ Since submittal of this Note, Miller and Carlson[8] have published a similar analysis. Their findings are also similar to to those presented previously.

Subsonic Lift Interference in a Wind Tunnel with Perforated Walls

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Introduction

THE development of large, high subsonic speed aircraft requires rather accurate wind-tunnel data for aerodynamic des'gn. Testing for such aircraft has normally been performed in transonic wind tunnels that have ventilated test sections such as the transonic wind tunnels at Arnold Engineering Development Center. Historically, the perforated walls were developed to minimize blockage interference at low supersonic Mach numbers by using a nonlifting, cone-cylinder model.[1] As a result, corrections to the data for a lifting-body model are necessary to obtain accurate data. This fact was pointed out in a recent study of wind-tunnel data correlation[2] where it was concluded that wind-tunnel wall effects must be taken into account to improve the quality of test data.

It is indicated in Ref. 3 that there is no available analytical solution for the boundary interference in a wind tunnel with perforated walls. The only existing solution has been obtained by an electrical analog measurement[4] for the case of a square tunnel with four walls perforated.

The purpose of this Note is to present an analytical solution of the boundary interference for wind tunnels with perforated walls. The method used in the calculation is the

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